Supplementary Information for

Investigating a corrective online measurement method for the tool influence function in millimetre spot-sized ion beam figuring

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Mathematical derivations for calculating the amplitude reduction ratio A_t and FWHM expansion ratio σ_t

According to the FC scanning procedure, the convolution effect can be mathematically calculated as follows:

$$t(\mu,\delta)=\begin{cases}1, \mu^2+\delta^2\leq(d/2)^2\\0, \mu^2+\delta^2>(d/2)^2\end{cases} \tag{1}$$

$$\begin{aligned}J_{\text{measure}}(x,0)&=\iint J(x-\mu,y-\delta)t(\mu,\delta)d\mu d\delta\\&=\iint J_{\max}\cdot\exp\left(-\frac{(x-\mu)^2+(0-\delta)^2}{2\sigma^2}\right)\cdot t(\mu,\delta)d\mu d\delta\end{aligned} \tag{2}$$

By transforming the above into the polar form,

$$J_{\text{measure}}(x,0)=J_{\max}\int_0^{2\pi}\int_0^{\frac{d}{2}}r\cdot\exp\left(-\frac{x^2+r^2-2xr\cos\theta}{2\sigma^2}\right)drd\theta \tag{3}$$

To calculate A_t, the value of x is equal to 0, and

$$J_{\text{measure}}(0,0)=J_{\max}\int_0^{2\pi}\int_0^{\frac{d}{2}}\exp\left(-\frac{r^2}{2\sigma^2}\right)\cdot rdrd\theta \tag{4}$$

According to the definition of the amplitude reduction ratio A_t, it can be calculated as follows:

$$A_t=\int_0^{2\pi}\int_0^{\frac{d}{2}}\frac{4}{\pi d^2}r\cdot\exp\left(-\frac{r^2}{2\sigma^2}\right)drd\theta \tag{5}$$

It is assumed that,

$$u=-\frac{r^2}{2\sigma^2} \tag{6}$$

$$du=-\frac{r}{\sigma^2}dr \tag{7}$$

Therefore, the amplitude reduction ratio A_t is finally determined as,

$$A_t=2\pi\int_0^{-\frac{d}{8\sigma^2}}\frac{4}{\pi d^2}\cdot e^u\left(-\sigma^2du\right)=\frac{8\sigma^2}{d^2}\left(1-e^{-\frac{d^2}{8\sigma^2}}\right) \tag{8}$$

To calculate the FWHM expansion ratio σ_t , the lateral position is defined with the expanded FWHM value,

$$C = \frac{\sigma_t \cdot 2\sqrt{2\ln 2} \cdot \sigma}{2} = \sigma_t \cdot \sqrt{2\ln 2} \cdot \sigma \tag{9}$$

Combined with the definition of the amplitude reduction ratio A_t ,

$$\frac{A_t}{2} = \frac{4}{\pi d^2} \int_0^{2\pi} \int_0^{\frac{d}{2}} r \cdot \exp\left(-\frac{C^2 + r^2 - 2r \cdot C \cdot \cos\theta}{2\sigma^2} \right) dr d\theta \tag{10}$$

After expansion,

$$\frac{A_t}{2} = \frac{4}{\pi d^2} \int_0^{2\pi} \int_0^{\frac{d}{2}} r \cdot \exp\left(-\frac{r^2 - 2 \cdot r \cdot \sigma_t \sqrt{2\ln 2} \cdot \sigma \cdot \cos\theta + \left(\sigma_t \cdot \sqrt{2\ln 2} \cdot \sigma\right)^2}{2\sigma^2} \right) dr d\theta \tag{11}$$

For integral terms with θ,

$$\int_0^{2\pi} \exp\left(\frac{-2 \cdot r \cdot \sigma_t \sqrt{2\ln 2} \cdot \sigma \cdot \cos\theta}{2\sigma^2} \right) d\theta = 2\pi I_0\left(\frac{\sqrt{2\ln 2} \cdot r \cdot \sigma_t}{\sigma} \right) \tag{12}$$

Thus, the following formula can be acquired:

$$\int_0^{d/2} \left(\frac{8r}{d^2} \right) \cdot \exp\left(-\frac{\left(2\ln 2\sigma_t^2 \sigma^2 + r^2\right)}{2\sigma^2} \right) \cdot I_0\left(\frac{\sqrt{2\ln 2} r \sigma_t}{\sigma} \right) dr = \frac{A_t}{2} \tag{13}$$

After simplification,

$$\frac{8}{d^2} \cdot e^{-\ln 2\sigma_t^2} \int_0^{\frac{d}{2}} r \cdot e^{\frac{-r^2}{2\sigma^2}} \cdot I_0\left(\frac{\sqrt{2\ln 2} \cdot r \cdot \sigma_t}{\sigma} \right) dr = \frac{A_t}{2} \tag{14}$$

Now, the value of A_t is known. Hence, the iterative method can be used to calculate the value of σ_t (e.g., using the function **FindRoot** in Mathematica).