Supplementary Information for 'The application of digital holography for accurate three-dimensional localisation of mosquito-bednet interaction'

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Abstract

The simulation described in the main manuscript involve modeling forward scattering of a mosquito up to the bednet, blocking and transmission of this wavefront at the bednet interface, and propagation of the combined wave to the detector plane. The combined wave propagates through a two-component telecentric demagnification system before being resized on the CCD/CMOS to simulate the sampling rate of an effective pixel size of interest. The images are also converted to an intensity in the range [0 4095] to simulate a 12-bit quantised intensity CCD/CMOS array. The following mathematical formulae explain the process in detail and how the simulated holograms at the detector were modelled.

1 Forward-Propagation Simulation

As discussed in the main manuscript, the mosquito can be considered as a transmission function $T_M[m,n] = 0$ for all points M corresponding to the mosquito and $T[m,n] = 1 \ \forall\ [m,n] \notin M$, where m and n are the rows and columns of the matrix in the spatial domain. An in-focus digital image of a mosquito was taken and binarised to yield the transmission function and digital representation of a mosquito for the simulations. The mosquito object to be propagated could therefore be considered as an incident wavefront $E_{i,M}[m,n]$ to the rest of the optical system or the bednet. The discretised wavefront propagation formulae of this wavefront using a Fresnel Transfer Function [1] is given by:

$$E_{O,M}[m,n] = \mathrm{DFT_{2D}}\left\{\mathrm{IDFT_{2D}}\left\{E_{i,M}[m,n]\right\} \times \mathcal{H}[p,q]\right\} \quad (1)$$

where DFT_{2D} and IDFT_{2D} are two-dimensional forward and inverse discretised fourier transform operations, p and q are the rows and columns in the frequency domain, and the transfer function $\mathcal{H}[p,q]$ is given by:

$$\mathcal{H}[p,q] = \exp[-jk_0 z] \exp\left[-j\pi\lambda z\left(\left(\frac{p}{R\Delta_x}\right)^2 + \left(\frac{q}{S\Delta_y}\right)^2\right)\right] \quad (2)$$

where R and S are the horizontal and vertical lengths of the matrix (in pixels) and Δ_x and Δ_y are the horizontal and vertical distances between each pixel, respectively.

The important variable to note is z, which is the propagation distance. In the first instance, z is the distance between the mosquito and the bednet, which was varied in the main manuscript to simulate different mosquito-bednet distances. The wavelength and wavenumber (λ, k_0) and discretised sampling $(p, q, m, n, \Delta x, \Delta y)$ are determined and fixed by the optical setup to be simulated.

Similarly to the mosquito, the bednet can be defined as a transmission function where $T_B[m,n] = 0$ for all locations B corresponding to the nylon fibres and $T[m,n] = 1 \ \forall\ [m,n] \notin B$. Again, an in-focus image of a bednet was binarised to simulate the transmission function of this object.

The Hadamard product (element-wise multiplication) is calculated between the mosquito object wavefront $E_{O,M}[m,n]$ and the bednet transmission function $T_B(m,n)$ to simulate blocking of the incoming wavefront where the bednet strands exist, and transmission where the holes exist, shown by:

$$E_{O,MB}[m,n] = E_{O,M}[m,n] \odot T_B(m,n) \tag{3}$$

This combined mosquito-bednet wavefront $E_{O,MB}[m,n]$ is then propagated to the back-focal plane of the two-component telecentric demagnification system using Equation 1, except the z term in the chirp function is changed to correspond to the propagation distance between the bednet and the back-focal point of the telecentric demagnification system.

Propagation through the first thin lens or focusing mirror is given by:

$$E_{O,L}[p,q] = \frac{{\Delta_x}^2}{j\lambda f} \exp\left[jk_0 f\right] \mathrm{DFT}_{2\mathrm{D}}\left\{E_{O,MB}[m,n]\right\} \tag{4}$$

where f is the focal length of the optic. This operation is performaced twice to simulate propagation through a two-component telecentric system to yield the wavefront incident on the detector, $E_{O,D}[m,n]$.

The incident resulting wavefront on the detector is complex-valued, so calculating the Hadamard product between this matrix $E_{O,D}[m,n]$ and its complex-conjugate converts the complex wavefront into an intensity to simulate recording on an intensity-based CCD/CMOS device, given by:

$$I_D[m,n] = E_{O,D}[m,n] \odot E^*_{O,D}[m,n] \tag{5}$$

The image matrix $I_D[m,n]$ was resized using an averaging function to simulate the different effective pixel sizes in the simulations in the manuscript. This intensity matrix was then quantised in the range [0 4095] to simulate a 12-bit intensity recording on a CCD/CMOS array.

2 Back-Propagation

Back-propagation was performed in the reverse order of the forward propagation algorithms, with focal distances f and propagation distances z reversed (i.e. $-f, -z$). The recorded volume was mathematically reconstructed by back-propagating to multiple values of z in 2 mm increments, and focus metrics (as defined in the main manuscript) were used to determine the z-axis position(s) of the bednet and mosquito.

References

[1] Poon, T. C. Liu, J. P. Introduction to Modern Digital Holography with MATLAB. (Cambridge: Cambridge University Press, 2014).